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INTUITIONISTIC FUZZY SECOND MINIMUM SPANNING TREE FOR SOLVING TRANSPORTATION AND ASSIGNMENT PROBLEM WITH OPTIMAL SOLUTION

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Abstract
In this paper we introduce a intuitionistic fuzzy minimum spanning tree for solving transportation and assignment problem .We characterize the transportation problem as a bipartite graph and solved it iteratively. We solve some problem numerically and find its optimal solution .

Keywords: Intuitionistic Fuzzy Set, Spanning tree, Intuitionistic Triangular Fuzzy Number, Intuitionistic Fuzzy Transportation Problem, Intuitionistic Fuzzy Assignment Problem

1. INTRODUCTION

In graph theory spanning tree problem is the most important and fundamental concept. Atanassov[1] introduced the concept of Intuitionistic fuzzy set. Deng Feng Li et al. [4] proposed a ranking method for triangular intuitionistic fuzzy number. Basic arithmetic operations of Triangular Intuitionistic Fuzzy numbers such as addition, subtraction and multiplication are defined by S.Mahapatra and T.K.Roy in [9], by considering the six tuple number itself. Anita Pal and ArindamDey[2] , Prim's algorithm for solving minimum spanning tree problem in fuzzy environment. The determination of the optimal path tree was efficiently determined by Bellman[3], Ford[6] and Moore [8]. Hassan[7] Presents a new algorithm based sushisen[11] on the distance matrix to solve the least cost minimum spanning tree problem. T.A. De Almeida et.al.[10] an

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evolutionary approach to solve minimum spanning tree problem with fuzzy parameters. Applications of minimum weight spanning tree were found in Foulds[5].

This article provides the best solution to the intuitionistic fuzzy minimum spanning tree transportation and assignment problem. Representation of equilibrium and unbalanced transportation assignment problems in a bipartite graph.

2.Preliminaries

2.1 Intuitionistic Fuzzy Set

An intuitionistic fuzzy set $\tilde{A}^I$ in X is given by a set of ordered triples:

$\tilde{A}^I = \{ <x, \mu_{\tilde{A}^I}(x), \upsilon_{\tilde{A}^I}(x)>/x \in X\}$, where $\mu_{\tilde{A}^I}$, $\upsilon_{\tilde{A}^I}$: X→[0,1] are the function such that $0 \le \mu_{\tilde{A}^I}(x) + \upsilon_{\tilde{A}^I}(x) \le 1$ for all x∈X. For each x the numbers $\mu_{\tilde{A}^I}(x)$ and $\upsilon_{\tilde{A}^I}(x)$ represent the degree of membership and degree of non membership of the element x∈X to A⊂X respectively.

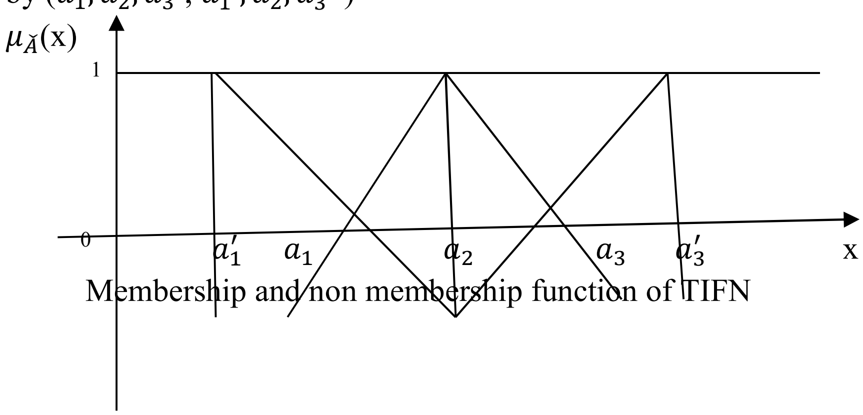
2.2 Triangular Intuitionistic Fuzzy Number (TIFN)

Let $\mu_{\breve{A}}$ is an intuitionistic fuzzy set in R with the following membership function $\mu_{\breve{A}}(x)$ and non membership function $\upsilon_{\breve{A}}(x)$:

$$\mu_{\breve{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & \text{for } a_1 \le x \le a_2 \\ \frac{a_3-x}{a_3-a_2} & \text{for } a_2 \le x \le a_3 \\ 0 & \text{otherwise} \end{cases}$$

$$\upsilon_{\breve{A}}(x) = \begin{cases} \frac{a_2-x}{a_2-a_1'} & \text{for } a_2 \le x \le a_3 \\ \frac{x-a_2}{a_3'-a_2} & \text{for } a_2 \le x \le a_3' \\ 1 & \text{otherwise} \end{cases}$$

Where $a_1' \le a_1 \le a_2 \le a_3 \le a_3'$ for all x∈X . Triangular intuitionistic fuzzy number $\breve{A}$ is denoted by $(a_1, a_2, a_3 \,;\, a_1', a_2, a_3')$



Membership and non membership function of TIFN

2.3 Defuzzification of Accuracy Function

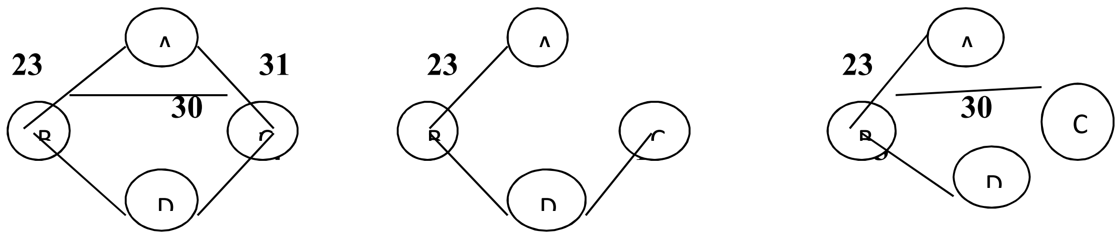
If $\breve{A} = (a_1, a_2, a_3 \,;\, a_1', a_2, a_3')$ is an Triangular Intuitionistic fuzzy number, then the defuzified value or the ordinary (crisp) number of $\breve{A}$,

$$\breve{A} = \frac{(a_1+2\,a_2+a_3)+(a_1'+2\,a_2+a_3'\,)}{8}$$

3.Fuzzy Second Minimum Spanning Tree (FSMST)

A Second Small value of Spanning Tree T* is a spanning tree , that has the second small value of all the edges, from all the possible spanning tree of the graph.

For example



FMST **FSMST**

3.1 Algorithm for Intuitionistic Fuzzy Second Minimum Spanning Tree Transportation(Balanced /Unbalanced) Problem

Step 1 : Find the crisp value of the given intuitionistic fuzzy transportation problem using ranking technique.

Step 2 : Examine whether total supply equals total demand . If not, introduce a dummy row or dummy column having all its cost elements as zero.

Step 3 : Convert the transportation table into bipartite graph.

Step 4 : Select the every edge with the second small value .

Step 5 : Select the edge from the bipartite graph which has maximum value of second minimum value select the spanning tree which has minimum cost coefficient.

Step 6 : Delete the source or demand vertex depending on which one is satisfied first.

Step 7 : Repeat from step 4 until there is no edge left

3.1 Example : 1

	D1	**D2**	**D3**	**D4**	**SUPPLY**
S1	(5,6,8); (2,6,9)	(2,3,4); (0,3,6)	(4,5,6); (2,5,8)	(6,2,3); (0,2,7)	(8,6,5) (2,6,9)
S2	(1,6,7); (0,6,8)	(7,8,9); (4,8,12)	(2,4,6); (0,4,7)	(6,10,10); (4,10,20)	(9,12,13); (4,12,14)
S3	(10,11,12); 1,11,21)	(11,15,16); (10,15,23)	(10,12,15); (7,12,16)	(10,13,14); (8,13,20)	(9,10,12); (6,10,13)
DEMAND	(3,5,5); (2,5,10)	(4,5,6); (3,5,7)	(2,6,10); (1,6,11)	(10,12,13); (9,12,16)	

By defuzzifying with the quantities

We get $A_{11} = 6$; $A_{12} = 3$; $A_{13} = 5$ Similarly , $A_{34} = 13$

	D1	D2	D3	D4	SUPPLY

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S1	6	3	5	2	6
S2	6	8	4	10	12
S3	11	15	12	13	10
DEMAND	5	5	6	12	

Convert the transportation table into bipartite graph

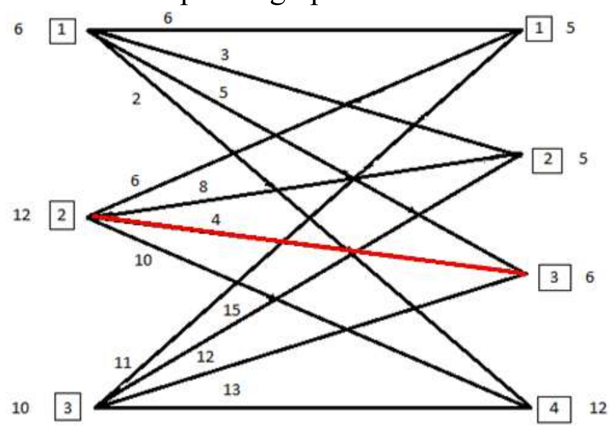

Figure (i)

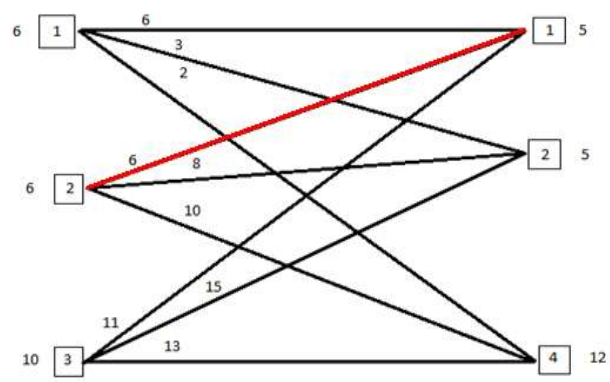

Figure (ii)

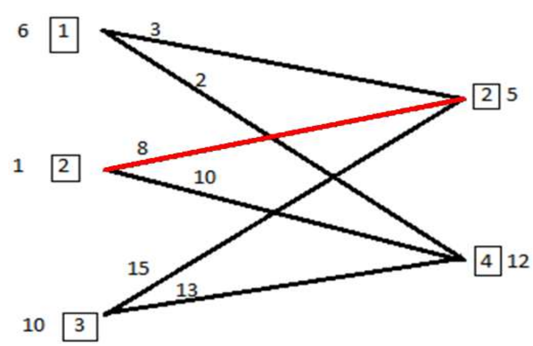

Figure (iii)

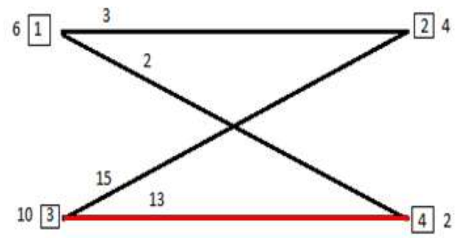

Figure (iv)

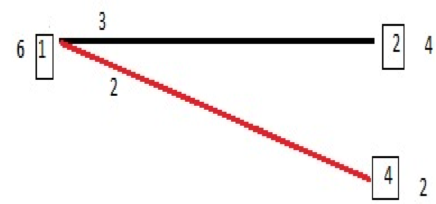

Figure (v)

Figure (vi)

$Z = 2\times2+3\times4+13\times10+8\times1+6\times5+4\times6$

$Z = 208$

3.1 Example : 2

	D1	**D2**	**D3**	**D4**	**SUPPLY**
S1	(6,9,10); (5,6,15)	(8,10,12); (6,10,14)	(6,8,9); (5,8,12)	(4,5,6); (2,5,8)	(20,30,40); (10,30,50)
S2	(2,4,6); (1,4,7)	(2,3,3); (1,3,6)	(4,6,5); (3,6,12)	(2,4,5); (1,4,8)	(10,20, 25); (5,20,40)
S3	(7,9,10); (6,9,13)	(3,5,6); (5,5,12)	(6,8,9); (5,8,12)	(1,2,3); (0,2,4)	(5,10,12); (3,10,20
DEMAND	(5,10,12); (3,10,20)	(20,30,30); (20,30,50)	(30,40,50); (20,40,60)	(5,10,15); (3,10,17)	

By defuzzifying with the quantities

We get $A_{11} = 9$; $A_{12} = 10$; $A_{13} = 8$ Similarly , $A_{34} = 2$

	D1	D2	D3	D4	SUPPLY
S1	9	10	8	5	30
S2	4	3	6	4	20

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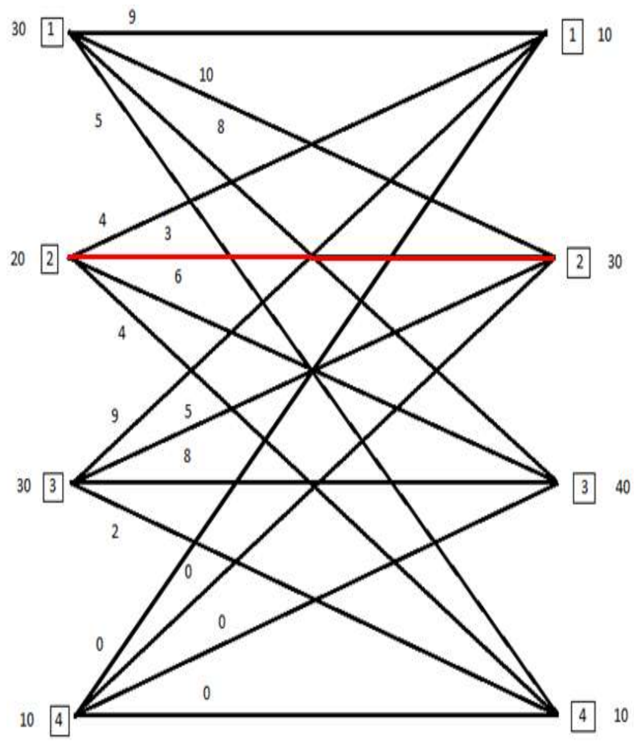
S3	9	5	8	2	30
DEMAND	10	30	40	10	

Unbalanced transportation table

	D1	D2	D3	D4	SUPPLY
S1	9	10	8	5	30
S2	4	3	6	4	20
S3	9	5	8	2	30
S4	0	0	0	0	10
DEMAND	10	30	40	10	

Convert the transportation table into bipartite graph



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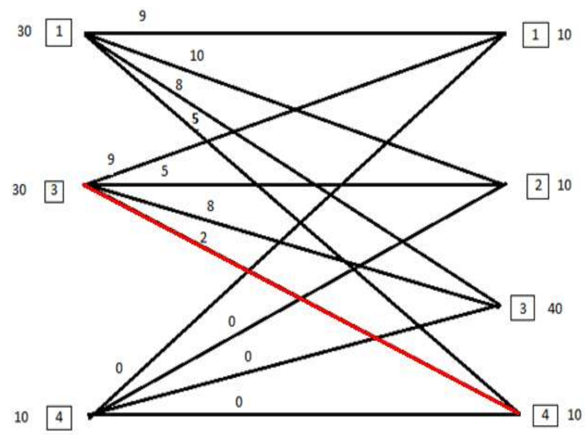
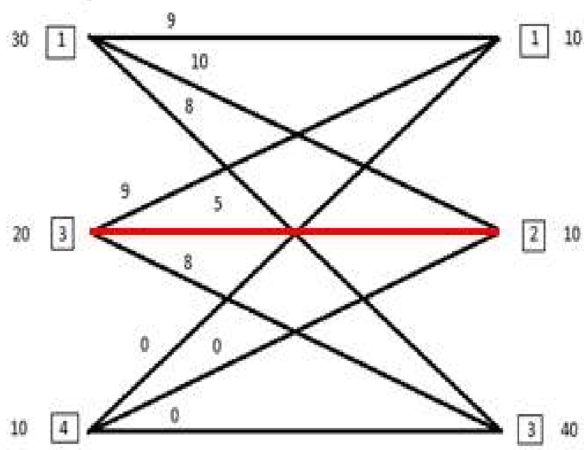
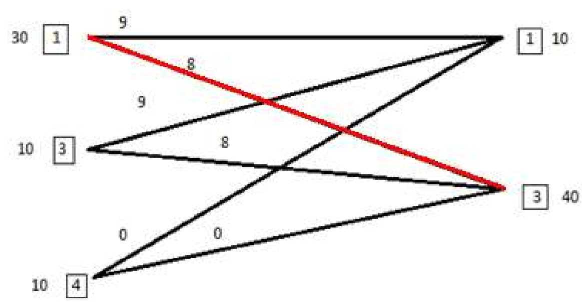
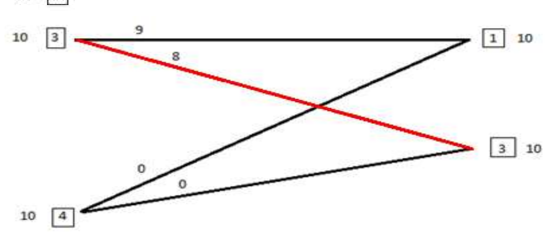








$Z = 8\times10+8\times30+5\times10+2\times10+3\times20$

$Z = 450$

4. Algorithm for Intuitionistic Fuzzy Second Minimum Spanning Tree Assignment(Balanced /Unbalanced) Problem

Step 1 : Find the crisp value of the given intuitionistic fuzzy assignment problem using ranking technique.

Step 2 : Examine whether the number of rows and columns in the cost matrix are equal . If not, introduce a dummy row(i) or dummy column(j) having all its cost elements as zero to form a square matrix.

Step 3 : Reduce each row element by the smallest element of that row redo the each column..

Step 3 : Convert the Assignment table into bipartite graph.

Step 4 : Select the every edge with the second small value .

Step 5 : Select the edge from the bipartite graph which has maximum value of second minimum value select the spanning tree which has zero cost coefficient.

Step 6 : Delete the i,j vertex into bipartite graph.

Step 7 : Repeat from step 4 until there is no edge left.

	Trichy (A)	Dindigul (B)	Mathurai(C)	Theni(D)
Passenger (I)	(6,10,13); (4,10,17)	(5,8,11); (3,8,13)	(14,15,18); (7,15,20)	(20,21,24); (10,21,30)
Passenger(J)	(4,6,9); (1,6,10)	(12,14,16); (9,14,19)	(2,3,4); (0,3,6)	(10,12,13); (7,12,18)
Passenger(K)	(9,11,13); (5,11,17)	(2,4,5); (1,4,8)	(3,7,9); (2,7,14)	(4,5,7); (6,5,11)
Passenger(L)	(15,16,17) (10,16,22)	(6,9,12); (3,9,15)	(10,13,14); (8,13,20)	(1,4,6); (0,4,7)

By defuzzifying with the quantities

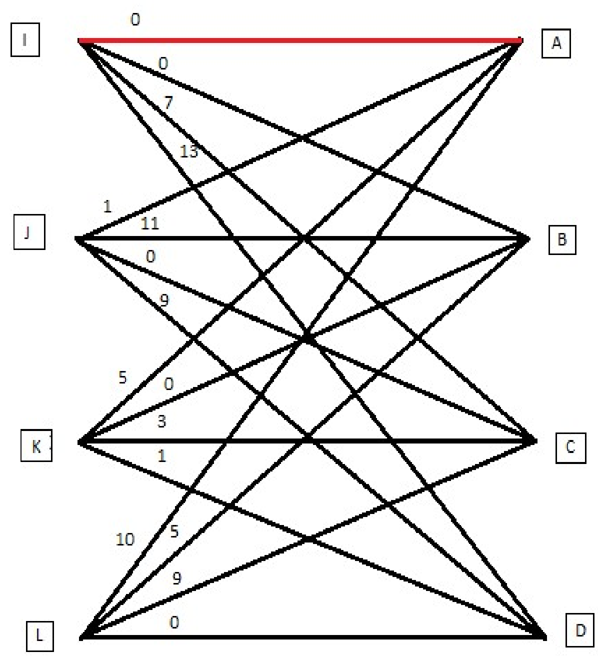
We get A_{11} = 10 A_{12} = 8 .... A_{44} = 4

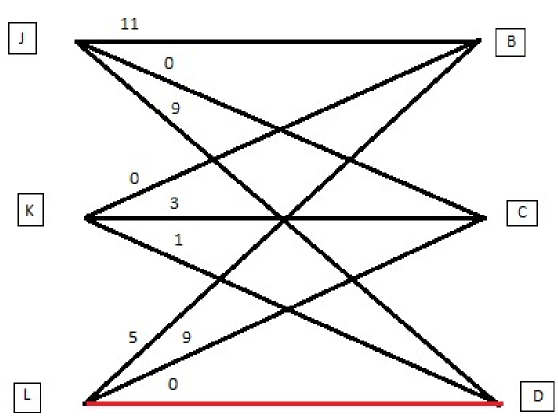
	A	B	C	D
I	10	8	15	21
J	6	14	3	12
K	11	4	7	5
L	16	9	13	4

	A	B	C	D
I	0	0	7	13
J	1	11	0	9

K	5	0	3	1
L	10	5	9	0

Convert the transportation table into bipartite graph





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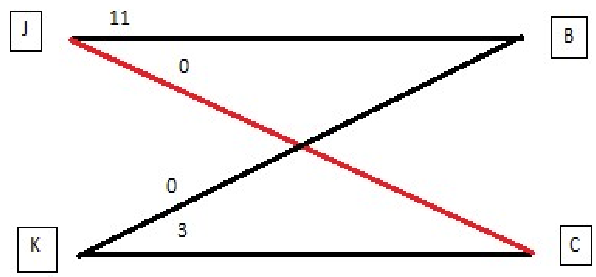

I → A , L→ D , J → C , K → B
The Optimal solution is 21 .

4.CONCLUSION

In this paper we have used a second minimum spanning tree graph theoretic approach to solve balanced and unbalanced intuitionistic fuzzy transportation and assignment problem . This method is based on the allocation of demand and supply items in the transportation matrix and find an optimal solution and ranking assignment .

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